Lesson 8 - Time Value of Money

Bell Ringer: Uncle Joe is giving you $100 for your Birthday. Would you rather have that money tomorrow or would you rather wait, let Uncle Joe invest it and get it 1 year from now? Why?

Instructors please read through the Take Home Reading to get some interesting ways to teach this material that sometimes is an abstract concept to students. There are some great resources there and great ways and nuisances that may help students remember.

A. Section 1 - Review Loans and Loan Payments

1. What is “interest” (cost of borrowing money)
2. what is “principal” (the money you actually borrow)
3. why do business managers or individuals get loans?
4. Calculate the monthly loan payment for a $10,000 car loan at 7.5% APR for 5 years

    Factor for 7.5% APR for 5 years is 20.04

    $10,000/$1000 x 20.04 = $200.40/month

B. Section 2 - Time Value of Money

1. People would rather have $10 today rather than waiting to get $10 sometime in the future
2. 3 main reasons:
    a. Risk – you may not receive that $10 in the future. This means you might settle for less money today rather than risking getting more money in the future. For example, would you rather have $90 in your hands today or wait 1 year to maybe receive $100? If you wait for 1 year, you may not receive any money at all.
    b. Inflation – inflation is the rise in the general price level
    As prices rise, that $10 will buy you less over time. So you can buy more if you have that $10 today. For example, if a gallon of milk costs $3.33/gal your $10 will buy you 3 gallons of milk. But if the price of milk increases to $4/gal you can only buy 2.5 gallons with your $10. Inflation is reducing your purchasing power, so you would rather have that money in your hands today so you can buy more with it.
    c. Opportunities – that $10 is more valuable to you today because you can do different things with it. You can:
        - Put it in your savings account
        - Spend it
        - Invest it for future goals
        - Pay down your debts
        - Donate it to church or charity

    You cannot do these things if you do not have the money in hand today. So, money in hand today is worth more to you because you can use it.
    d. For these 3 reasons, we say that money has a “time value” – money in hand today is worth more than the promise of money in the future!

C. Section 3 - Compound Interest

1. "Compounding" means that you are earning interest on top of interest that you have already earned. This is a powerful financial concept!

   Example: Assume you invest $1,000 (principal) in an account that earns 10% annual return.

   - After 1 year, you will earn $100 of interest ($1,000 x 10%)
     - You now have $1,100 in your account ($1,000 + $100)
     - From now on, this entire amount is treated as principal
   - After year 2, you will earn $110 of interest ($1,100 x 10%)
   - After year 3, you will earn $121 of interest ($1,210 x 10%) and so on…

D. Section 4 - There are several types of Time Value of Money problems to solve. The main ones are:

1. Future Value of a lump sum (a one-time investment)
   This estimates how much money you will have in the future if you invest money today
2. Present Value of a lump sum (a one-time investment)
   This is used when you are supposed to receive money in the future (one time). We use Present Value to estimate how much you would rather have in hand today vs waiting to hopefully receive that promised amount in the future.
3. Future Value of a stream of investments (annuities)
   An annuity is a stream of constant payments over time. Monthly car loan payments are an example of an annuity – you make the same payment every month for a given number of years. We use Future Value of Annuities to estimate how much money you will have in your account if you make regular investments into that account over time. For example, how much money you will have in your account if you invest $100/month for the next 5 years.
4. We can solve these problems in several ways:
   a. Time Value of Money tables
   b. Spreadsheets
   c. Financial Calculators
   d. Formulas
5. We will focus on Time Value of Money tables and spreadsheets

E. Section 5 - Future Value of a Lump Sum

1. A "lump sum" is a one-time investment – you only pay or receive the money one time
   For example, assume that you deposit $1,000 today into an investment that earns a 5% rate of return ("it pays 5%"). The $1,000 is a lump sum since you are only investing the money one time.
2. Future Value means that we are trying to find out how much your money will be worth in the future.
3. To calculate how much money you will have in your account after 5 years, we will calculate the Future Value of this Lump Sum
   - Using the Time Value Tables, go to Table 1 "Future Value of Lump Sum Factors"
   - Step 1: Find the Future Value factor from Table 1
     - Use this table just like you used the annual loan payment tables
     - Look across the top row to find the 5% column
     - Go down that column until you reach the "n = 5" row.
     - The factor for 5% for five years is 1.2763
   - Step 2: Multiply the lump sum by the factor
     - $1,000 x 1.2763 = $1,276.30

- If you invest $1,000 today in an investment that earns a 5% rate of return, your money will grow to $1,276.30 over the next 5 years. That is, you earned $276.30 ($1,276.30 - $1,000) of returns by just letting your money sit in the account.
- What will your account grow to after 40 years?
  Factor for 5% for 40 years = 7.0400
  $1,000 x 7.0400 = $7,040 at the end of 40 years
  Your initial $1,000 deposit grew to $7,040 over these 40 years

Work some other problems of your own – or work through the Future Value problems on the exercise to give the students practice using the tables

F. Section 6 - Present Value of a Lump Sum

1. Present Value calculates how much money you need to invest today to reach a future goal. We can also use Present Value to determine how much money you would rather have in your hands today rather than waiting to hopefully receive money in the future (you may not receive it due to risk!)
2. This is basically the "opposite" of Future Value
3. We solve Present Value problems in a very similar manner to Future Value
   Example: You want to have $10,000 in your account after 5 years so that you can make a down payment on a piece of land. Your account earns a 6% rate of return.
   - Step 1. Find the Present Value factor from Table 2
     - Factor for 6% for 5 years is 0.7473
   - Step 2 . Multiply the lump sum by the factor
     - $10,000 x 0.7473 = $7,473
     So, if you invest $7,473 today in an account that pays 6% per year, you will have $10,000 in the account after 5 years.

Work some other problems of your own – or work through the Present Value problems on the exercise to give the students practice using the tables

G. Section 7 - Future Value of an Annuity (stream of payments)

1. An annuity is a regular stream of payments – like monthly car loan payments.
2. We use Future Value of an Annuity to estimate how much money will be in your account after a certain time period if you are making regular investments during that time period.
   For example – assume that you want to invest $1,000/year in an account that earns 7% per year. How much will you have in your account after 30 years?
     - You might think, "I'm investing $1,000/year for 30 years – I should have around $30,000."
     - But don't forget about the earnings on your account!!
   - Step 1. Find the Future Value of Annuity factor from Table 6
     - Factor for 7% for 30 years is 101.0730
   - Step 2. Multiply the Annuity ($1,000/year) by the factor
     - $1,000 x 101.0730 = $101,073
     So, you invested a total of $30,000 of principal into your account over time ($1,000/yr x 30 years). But you end up with over $100,000!! This is due to the compound earnings on the account. This is a powerful tool for business managers!

H. Section 8 - Using a Spreadsheet for Time Value of Money Calculations

- Spreadsheets are great tools for time value of money calculations. All you need to do is enter your information and let the spreadsheet do the calculations.
- Using the Future Value Calculator
  - Enter your data in the cells with the BLUE font
  - Example: You want to invest $300/year into an account that earns 6%. How much will you have in your account after 5 years?

Number of Years	5		Cell C3
Annual Rate	6		Cell C7
Payment to be Made	300		Cell C9
Present Value (Lump Sum)	0		Cell C10
Future Value		$1,792.60	

What if you wanted to look at an annual return of 10%?

- Simply change the 6 to 10 in the Annual Rate cell (C7)
  - At 10%, the Future Value is $2,014.68

I. Section 9 - Using the Present Value Calculator

- Click on the tab named “PV Calculator” at the bottom of the spreadsheet
- You’ll see a Present Value Calculator very similar to the Future Value Calculator
- Enter your data in the cells with the BLUE font
- Example: You want to have $40,000 at the end of 10 years. Your account that earns 7%. How much will you need to invest today (lump Sum) to reach your goal?

Number of Years	10		Cell C3
Annual Rate	7		Cell C7
Payment to be Made	0		Cell C9
Future Value (Lump Sum)	40,000		Cell C10
Present Value		$20,333.97	

What if you wanted to look at an annual return of 10%?

- Simply change the 6 to 10 in the Annual Rate cell (C7)
  - At 10%, the Present Value is $15,421.73

Materials: **PowerPoint on Time Value of Money**
Note Organizer
In-class Exercise and Key
Homework Exercise and Key
Student Driven Learning Activity
Take Home Reading

TIME VALUE OF MONEY

$10 Today vs. $10 Next Year?

- Most people would rather have $10 today rather than waiting to be paid $10 next year
- 3 main reasons:
  - *Risk*
    - You may not get paid in the future!
  - *Inflation*
    - As prices increase, that $10 will buy less in the future
  - Opportunities
    - You can do something with that $10 today
      - *Save, pay down loans, invest, spend, donate, etc.*

Time Value of Money

- Having money in hand today is more valuable than waiting to maybe receive money in the future
  - *RIO (Risk, Inflation, Opportunities)*
- This is called the "time value of money"

Compound Interest

- Powerful financial tool!
- Compounding means:
  - *"Earning interest on top of interest"*
  - *The interest you earn in period 1 will earn interest in period 2…*
- Example: You invest $1,000 today in an account that earns 10% annual return
  - *How much will you earn over the next 3 years?*

Compound Interest

- Year 1: $1,000 x 10% = $100 of interest
- Year 2: ($1,000 + $100) x 10% = $110 of interest
  - *Notice you earned $10 more dollars of interest in Year 2*
    - The $100 of interest in Year 1 is treated as principal for the Year 2 calculation
- Year 3: ($1,000 + $100 + $110) x 10% = $121 of interest

Compound Interest

- Year 1 = $100 earned
- Year 2 = $110 earned
- Year 3 = $121 earned
- Total interest earned = $331
- If you earned "simple interest" you would only earn $300 of interest
  - *$1,000 x 10% x 3 years = $300*

Terms

- Lump Sum = a one-time investment
  - *Ex. You invest $500 today and invest nothing else after that*
- Annuity = stream of regular payments
  - *Ex. Car loan payments – they are the same amount every month for a stated number of years*
- Future Value = what you will have in your account in the future
- Present Value = what something is worth today

Types of Time Value Problems

- Future Value of a Lump Sum
  - *Determines how much money an investment will be worth in the future if you invest money today*
- Present Value of a Lump Sum
  - *Determines how much you would rather have today instead of waiting to be paid (maybe) in the future*
  - *Also, it determines how much you need to invest today to reach a specific future value*

Types of Time Value Problems

- Future Value of an Annuity
  - *Determines how much you will have in your account in the future if you invest regularly over time*
  - *Example: You invest $500/year into a retirement account that earns 8% return. How much will you have in your account after 50 years?*

Solving Time Value Problems

- 4 methods:
  - *Time Value of Money tables*
  - *Excel spreadsheets*
  - *Financial calculators*
  - *Time Value of Money formulas*
  - *We will focus on the tables and spreadsheets*

Using the Time Value Tables

- Same as the annual loan payment table
- Look up the column with the interest rate
- Follow the column down to the row with the number of years
- Multiply that factor by the dollar amount of the investment

Future Value of a Lump Sum

- Use Table 1
- Example: You invest $1,000 today in an account that earns 5%. How much will you have in your account after 5 years?
- Factor for 5% for 5 years = 1.2763
- FV = 1.2763 x $1,000 = <u>$1,276.30</u>
  - *Your $1,000 grew to almost $1,300 in 5 years!!*

Future Value of a Lump Sum

- Assume that you leave your money in the account for 40 years – you do Not add any more money. How much will you have in your account after 5 years at a 5% return?
- Factor for 5% for 40 years = 7.0400
- FV = 7.0400 x $1,000 = $7,040.00
  - *Your initial investment of $1,000 grew to more than $7,000!!*

Present Value of a Lump Sum

- Use Table 2
- PV is the "opposite" of FV
- Example: You want to have $10,000 available after 5 years for a down payment on some land. How much do you need to invest today to reach this goal at a 6% return?
- Factor for 6% for 5 years = 0.7473
- PV = 0.7473 x $10,000 = $7,473

Present Value of a Lump Sum

- From this example:
  - *If you invest $7,473 today*
  - *It earns 6% each year (compound interest)*
  - *It will grow to $10,000 in 5 years*

Future Value of an Annuity

- Use Table 6
- Example: You invest $1,000/yr for 30 years. It earns 7% return. How much will you have after 30 years?
  - *You might think somewhere around $30,000*
    - $1,000/yr x 30 yr = $30,000
- Factor for 7% for 30 years = 101.0730
- FV = $101.0730 x $1,000/yr = $101,073
  - *That's a lot more than the $30,000 you invested!!*

Using a Time Value Spreadsheet

- Spreadsheets are great for analysis!
  - *You can change key factors to see the impact*
- For Future Value calculations
  - *Click on the "FV Calculator" tab*
  - *You can change any number with a blue font*
    - It will automatically recalculate the FV

Future Value Spreadsheet

- Example: You want to invest $300/yr at 6%. What will you have after 5 years?
  - *Number of Years Cell C3 Enter 5*
  - *Annual Rate (%) Cell C7 Enter 6*
  - *Annuity Cell C9 Enter 300*
  - *Present Value Cell C10 Enter 0*
  - *FV = $1,792.60*
  - *What is your FV if you earn 10% instead of 6%?*
    - Simply change Cell C7 (Annual Rate) to 10

Present Value Spreadsheet

- Click on the "PV Calculator" tab
- Use the same as the FV calculator
- Example: You want to have a future value of $40,000 after 10 years. How much do you need to invest today earning 7%?
  - *Years = 10*
  - *Annual Rate = 7%*
  - *Annuity = 0*
  - *FV (Lump Sum) = 40,000*
  - *PV = $20,333.97*

Keep in Mind

- Lump sum = only investing 1 time
- Annuity = several constant investments
- If you know the FV, solve for the PV
- If you know the PV, solve for the FV
- If it helps, draw a timeline
  - *This can help you figure out what to solve for!*

Time Value of Money- Note Organizer

$10 Today vs. $10 Next Year?

- Most people would rather have $10 today rather than waiting to be paid $10 next year
- 3 main reasons:
  - ________________
    - You may not get paid in the future!
  - Inflation
    - As prices increase, that $10 will buy less in the future
  - ________________
    - You can do something with that $10 today
      - Save, pay down loans, invest, spend, donate, etc.

Time Value of Money

- Having money in hand today is more valuable than waiting to maybe receive money in the future
  - RIO (Risk, Inflation, Opportunities)
- This is called the "________________________"

Compound Interest

- Powerful financial tool!
- ________________________________:
  - "Earning interest on top of interest"
  - The interest you earn in period 1 will earn interest in period 2…
- Example: You invest $1,000 today in an account that earns 10% annual return
  - How much will you earn over the next 3 years?
    - Year 1: $1,000 x 10% = $100 of interest
    - Year 2: ($1,000 + $100) x 10% = $110 of interest
      - Notice you earned $10 more dollars of interest in Year 2
      - The $100 of interest in Year 1 is treated as principal for the Year 2 calculation
    - Year 3: ($1,000 + $100 + $110) x 10% = $121 of interest
    - Compound Interest
      - Year 1 = $100 earned
      - Year 2 = $110 earned
      - Year 3 = $121 earned
      - Total interest earned = $331
    - If you earned "simple interest" you would only earn $300 of interest
      - $1,000 x 10% x 3 years = $300

Terms

- ________________ = a one-time investment
  - Ex. You invest $500 today and invest nothing else after that
- Annuity = stream of regular payments
  - Ex. Car loan payments – they are the same amount every month for a stated number of years
- _______________ = what you will have in your account in the future
- Present Value = what something is worth today

Types of Time Value Problems

- ____
  - Determines how much money an investment will be worth in the future if you invest money today
- Present Value of a Lump Sum
  - Determines how much you would rather have today instead of waiting to be paid (maybe) in the future
  - Also, it determines how much you need to invest today to reach a specific future value
- ____
  - Determines how much you will have in your account in the future if you invest regularly over time
  - Example: You invest $500/year into a retirement account that earns 8% return. How much will you have in your account after 50 years?

Solving Time Value Problems

- 4 methods:
  - Time Value of Money tables
  - __________________________
  - Financial calculators
  - Time Value of Money formulas

    **We will focus on the tables and spreadsheets

Using the Time Value Tables

- Same as the annual loan payment table
- Look up the column with the _______________________
- Follow the column down to the row with the number of years
- Multiply that factor by the _________________________ of the investment

Future Value of a Lump Sum

- Use Table 1
- Example: You invest $1,000 today in an account that earns 5%. How much will you have in your account after 5 years?
  - Factor for 5% for 5 years = 1.2763
  - FV = 1.2763 x $1,000 = <u>$1,276.30</u>

- Your $1,000 grew to almost $1,300 in 5 years!!

- Assume that you leave your money in the account for 40 years – you do NOT add any more money. How much will you have in your account after 5 years at a 5% return?
  - Factor for 5% for 40 years = 7.0400
  - FV = 7.0400 x $1,000 = $7,040.00
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Present Value of a Lump Sum

- Use Table 2
- PV is the "_____________" of FV
- Example: You want to have $10,000 available after 5 years for a down payment on some land. How much do you need to invest today to reach this goal at a 6% return?
  - Factor for 6% for 5 years = 0.7473
  - PV = 0.7473 x $10,000 = $7,473
- From this example:
  - If you invest $7,473 today
  - It earns 6% each year (compound interest)
  - It will grow to $10,000 in 5 years

Future Value of an Annuity

- Use Table 6
- Example: You invest $1,000/yr for 30 years. It earns 7% return. How much will you have after 30 years?
  - You might think somewhere around $30,000
    - $1,000/yr x 30 yr = $30,000
  - Factor for 7% for 30 years = 101.0730
  - FV = $101.0730 x $1,000/yr = $101,073
    - That's a lot more than the $30,000 you invested!!

Using a Time Value Spreadsheet

- ____!
  - You can change key factors to see the impact
- For Future Value calculations
  - Click on the "__________________________" tab
  - You can change any number with a blue font
    - It will automatically recalculate the FV

- Future Value Spreadsheet
- Example: You want to invest $300/yr at 6%. What will you have after 5 years?
  - Number of Years Cell C3 Enter 5

- Annual Rate (%) Cell C7 Enter 6
- Annuity Cell C9 Enter 300
- Present Value Cell C10Enter 0

- FV = $1,792.60
- What is your FV if you earn 10% instead of 6%?
  - Simply change Cell C7 (Annual Rate) to 10

- Present Value Spreadsheet
  - Click on the "PV Calculator" tab
  - Use the same as the FV calculator
  - Example: You want to have a future value of $40,000 after 10 years. How much do you need to invest today earning 7%?
    - Years = 10
    - Annual Rate = 7%
    - Annuity = 0
    - FV (Lump Sum) = 40,000
    - PV = $20,333.97

Keep in Mind

- Lump sum = only investing 1 time
- ____________ = several constant investments
- If you know the FV, ____________________
- If you know the PV, solve for the FV

- If it helps, draw a timeline
  - This can help you figure out what to solve for!

Time Value In-Class Exercise

1. You deposit $1,000 in a mutual fund (a one-time deposit) that earns 8% compounded annually.

    a. How much will you have in your account at the end of 10 years?

    b. At the end of 40 years?

    c. What if you had invested the $1,000 in a savings account that earned 2% annually – how much would you have in your account after 40 years?

2. You just bought a plot of land for $4,000/acre in hopes that it will increase in value 7% each year.

    a. How much will the land be worth in 10 years assuming it increases in value by 7% each year?

    b. How much will it be worth after 40 years?

3a. You want to have $50,000 at the end of 10 years in order to make a down payment on your business. How much do you need to invest today (today only), earning 8% per year, to have $50,000 in your account after 10 years?

3b. Similar to 3a, you want to have $50,000 at the end of 10 years. You can invest $4,000/year for each of the next 10 years. Your investment will earn a return of 8% per year. Will you be able to reach your goal?

4. You want to contribute $5,000/year to an IRA (Individual Retirement Account) – investing in assets that earn about 8 percent annually. How much will you have in the IRA after 20 years? 40 years?

5. The average America family has annual living expenses (food, rent, utilities, etc.) of $50,000 per year. Let's assume that the annual inflation rate is 3% per year. How much will it cost an average American family to have the same level of living ($50,000/year) 50 years from now? (50 years is approximately when you will be retiring!)

Time Value In-Class Exercise (Key)

1. You deposit $1,000 in a mutual fund (a one-time deposit) that earns 8% compounded annually.

   a. How much will you have in your account at the end of 10 years?

      FV Factor (Table 1) for 8% for 10 years = 2.1589
      $1,000 x 2.1589 = $2,158.90   Your money more than doubled in 10 years!

   b. At the end of 40 years?

      FV Factor (Table 1) for 8% for 40 years = 21.7245
      $1,000 x 21.7245 = $21,724.50   Your $1,000 grew to almost $22,000!!

   c. What if you had invested the $1,000 in a savings account that earned 2% annually – how much would you have in your account after 40 years?

      FV Factor (Table 1) for 2% for 40 years = 2.2080
      $1,000 x 2.2080= $2,208 Your $1,000 only grew to $2,200. Compare this result to part c. where it grew to $22,000!!

2. You just bought a plot of land for $4,000/acre in hopes that it will increase in value 7% each year.

   a. How much will the land be worth in 10 years assuming it increases in value by 7% each year?

      FV Factor (Table 1) for 7% for 10 years = 1.9672
      $4,000 x 1.9672= $7,868.80

   b. How much will it be worth after 40 years?

      FV Factor (Table 1) for 7% for 40 years = 14.9745
      $4,000 x 14.9745= $59,898

3a. You want to have $50,000 at the end of 10 years in order to make a down payment on your business. How much do you need to invest today (today only), earning 8% per year, to have $50,000 in your account after 10 years?

   This is a Present Value of a Lump Sum problem
      PV Factor (Table 2) for 8% for 10 years = 0.4632

      $50,000 x 0.4632 = $23,160

   If you invest $23,160 today, it will grow to $50,000 at the end of 10 years if you earn an annual return of 8%.

3b. Similar to 3a, you want to have $50,000 at the end of 10 years. You can invest $4,000/year for each of the next 10 years. Your investment will earn a return of 8% per year. Will you be able to reach your goal?

This is a Future Value of an Annuity problem

FVA Factor (Table 6) for 8% for 10 years = 15.6455

$4,000 x 15.6455= $62,582

Yes, if you invest $4,000/yr for the next 10 years, it will grow to $62,582. This is greater than the $50,000 you wanted to have.

4. You want to contribute $5,000/year to an IRA (Individual Retirement Account) – investing in assets that earn about 8 percent annually. How much will you have in the IRA after 20 years? 40 years?

This is a Future Value of an Annuity problem

FVA Factor (Table 6) for 8% for 20 years = 49.4229

$5,000 x 49.4229= $247,115 (but you only invested $100,000 over this time!)

FVA Factor (Table 6) for 8% for 40 years = 279.7810

$5,000 x 279.7810 = $1,398,905 (but you only invested $200,000 over this time!)

5. The average America family has annual living expenses (food, rent, utilities, etc.) of $50,000 per year. Let's assume that the annual inflation rate is 3% per year. How much will it cost an average American family to have the same level of living ($50,000/year) 50 years from now? (50 years is approximately when you will be retiring!)

This is a Future Value of a Lump Sum question

FV Factor (Table 1) for 3% for 50 years = 4.3839

$50,000 x 4.3839 = $219,195 (it will cost over 4 times as much for the same level of living – just due to inflation!!)

Time Value of Money Homework Exercise

Use the Time Value of Money tables to answer the following questions. Show your work!

1. You just purchased a house for $130,000. Similar houses in your area are going up in value at a rate of 5% per year.

    a. How much will your house be worth at the end of 15 years?

    b. How much will it be worth at the end of 30 years?

2. Your elderly neighbor just told you that he purchased his first new car for $1,500 about 50 years ago. That has you wondering how much a new car will cost you when you are older. Car prices today average $20,000. It appears that car prices increase at a rate of 6% every year. How much will a new car cost 50 years from today?

3. You just won a prize!! The company that sponsored the prize will pay you $4,000, but you won't get this $4,000 until 3 years from today. Rather than waiting 3 years to collect this money, you are thinking of selling your rights to this prize to someone else so that you will receive some cash today. You can earn a return of 8% on your money. What is the lowest amount of money that you would sell your rights to this prize?

4. What are the three main reasons that money has a time value?

5. Your church wants to build a new community education center, so they have set a goal of collecting $250,000 over the next 8 years to pay for the building. They can invest their money in account that earns 5% each year. They hope to collect contributions of $25,000/year over the next 8 years. Will the church be able to reach their goal? (Assume BGN payments)

6. Your grandparents started investing for your college tuition as soon as you were born. They invested $2,000/year every year since you were born. Their college investment account earned a return of 7% each year. How much money will be in your college education account after 18 years of contributions? (Assume BGN payments)

7. Use the TVM Calculator spreadsheet to double-check your answers. List the answers to each question that you get from using the spreadsheet:

   Question 1: ______________________________

   Question 2: ______________________________

   Question 3: ______________________________

   Question 5: ______________________________

   Question 6: ______________________________

Time Value of Money Homework Exercise - KEY

Use the Time Value of Money tables to answer the following questions. Show your work!

1. You just purchased a house for $130,000. Similar houses in your area are going up in value at a rate of 5% per year.

   a. How much will your house be worth at the end of 15 years?

N = 15	**Table 1 – FV of a Lump Sum**
I = 5%	**Factor for 5%, 15 years = 2.0789**
PV = $130,000	
PMT = $0	**FV = $130,000 x 2.0789 = $270,257**
FV = ?? = $270,257	

   b. How much will it be worth at the end of 30 years?

N = 30	**Table 1 – FV of a Lump Sum**
I = 5%	**Factor for 5%, 30 years = 4.3219**
PV = $130,000	
PMT = $0	**FV = $130,000 x 4.3219 = $561,847**
FV = ?? = $561,847	

2. Your elderly neighbor just told you that he purchased his first new car for $1,500 about 50 years ago. That has you wondering how much a new car will cost you when you are older. Car prices today average $20,000. It appears that car prices increase at a rate of 6% every year. How much will a new car cost 50 years from today?

N = 50	**Table 1 – FV of a Lump Sum**
I = 6%	**Factor for 6%, 50 years = 18.4202**
PV = $20,000	
PMT = $0	**FV = $20,000 x 18.4202 = $368,404**

   FV = ?? = $368,404 is the purchase price for a new car 50 years from now

3. You just won a prize!! The company that sponsored the prize will pay you $4,000, but you won't get this $4,000 until 3 years from today. Rather than waiting 3 years to collect this money, you are thinking of selling your rights to this prize to someone else so that you will receive some cash today. You can earn a return of 8% on your money. What is the lowest amount of money that you would sell your rights to this prize?

N = 3	**Table 2 – PV of a Lump Sum**
I = 8%	**Factor for 8%, 3 years = 0.7938**

   PV = ?? = $3,175.20 is the minimum price you would take

PMT = $0	**PV = $4,000 x 0.7938= $3,175.20**
FV = $4,000	

4. What are the three main reasons that money has a time value?
   Risk
   Inflation **RIO**
   Opportunity Cost

5. Your church wants to build a new community education center, so they have set a goal of collecting $250,000 over the next 8 years to pay for the building. They can invest their money in account that earns 5% each year. They hope to collect contributions of $25,000/year over the next 8 years. Will the church be able to reach their goal? (Assume BGN payments)

N = 8	**Table 7 – FV of an Annuity**
I = 5%	**Factor for 5%, 8 years = 10.0266**
PV = $0	
PMT = $25,000	**FV = $25,000 x 10.0266 = $250,665**
FV = ?? = $250,665	

Yes, they will be able to meet their goal because the FV is greater than the $250,000 goal.

6. Your grandparents started investing for your college tuition as soon as you were born. They invested $2,000/year every year since you were born. Their college investment account earned a return of 7% each year. How much money will be in your college education account after 18 years of contributions? (Assume BGN payments)

N = 18	**Table 7 – FV of an Annuity**
I = 7%	**Factor for 7%, 18 years = 36.3790**
PV = $0	
PMT = $2,000	**FV = $2,000 x 36.3790 = $72,758**
FV = ?? = $72,758	

Your college education account will have $72,758 after 18 years of contributions by your grandparents. Thank you, grandparents!

7. Use the TVM Calculator spreadsheet to double-check your answers. List the answers to each question that you get from using the spreadsheet:
   Question 1: **a. $270,260.66 b. $561,852.51**

   Question 2: **$368,403.09**

   Question 3: **$3,175.33**

   Question 5: **$250,664.11**

   Question 6: **$72,757.93**

Time Value of Money- Student Driven Activity

Student Driven Learning Activity: Have students complete the in class exercises together as a group with the teacher as the facilitator. Then have the students break into groups and complete the homework exercises together. After completing the exercises have each group come up and solve the exercise for the rest of the class. After completing that have each group come up with 3 scenarios/exercises on their own, make copies and hand out to the remainder of the class to solve. After completing all the exercises the original group will lead the rest of the class through how to come up with the answer to their scenario/exercise.

Lesson 8 – Time Value of Money Take-Home Reading

Stop and think for a moment – what is the most powerful thing on Earth? Depending on whom you ask, you will get a variety of answers. A scientist might answer with "water, or maybe wind, or the atom." A philosopher might say "time". A religious person might say "faith." An engineer will answer with, "leverage." But someone familiar with financial principles will undoubtedly answer that question with the phrase, "time value of money." But what exactly is time value of money (TMV)?

The phrase "time value of money" very simply refers to the fact that most people would rather at least some money in their hands today ("cash on the barrelhead") rather than waiting to possibly receive a larger amount of money in the future. That is, money has a different value depending on the time that you receive it. Ask yourself this question, "Would you rather have $90 of cash in your hands today or would you prefer to wait 1-2 years to receive $100?" Most people would rather have $100 than $90 because it's more money. The problem is that you have to wait to receive that $100. And what can happen during that waiting period?

There are three main reasons that you might prefer to have the $90 today instead of the $100 in the future:

- Risk
- Inflation
- Opportunity Cost

An easy way to remember these three factors is the acronym "RIO", as in Rio Grande. Risk is the easiest to remember – there is a risk that you will not receive the $100 in the future. The person with whom you made the deal may have left town, or simply forgotten, or possibly he is bankrupt and cannot pay you the $100. So, would you rather have $90 in your hands today or wait to maybe receive $100 in the future? You must think about the risks that are involved.

Inflation is a term that means that prices are increasing over time. Think about food prices. A cartful of groceries might cost $90 today. But one year from now it might cost $95. It's the same groceries, but they cost more. This is called inflation. If you only had $90 today you could purchase that cart of groceries. But 1 year from now if you had that same $90 you would not be able to purchase that cart of groceries. This means that $90 is more valuable to you today than $90 one year from today – you can buy more with it today because of the rising prices over time.

The last term, "Opportunity Cost" can be a little confusing at first, but it's a relatively easy concept. "Opportunity" means that you have possibilities for what you can do with that $90 if you have it in hand today. You can spend it on something (like groceries); you can save it for emergencies; you can use it to pay down one of your loans; you can give it to someone (church, charity, gifts); or you can invest it to try to have more money in the future. These are your "opportunities" if you have the cash in hand today. The "Cost" part of the term indicates that there is a cost for using that $90 for any of these opportunities. If you spend it, you cannot use it to pay down your debts or invest for the future. If you save it for emergencies, you can't take advantage of lower prices by spending it today; and so on. In a nutshell, "opportunity cost" is what you are giving up by using your money. Because you always want to use your money in the best way possible, we consider the opportunity cost to be the "next best use" of the money.

Here's an example for Opportunity Cost. You just received $1,000 as a graduation gift from your grandparents. To keep it simple, assume that you have the following options:

- keep it in your savings account where it earns 1% interest each year
- use it to pay down your car loan which has an interest rate of 5%

If you keep the money in your savings account, you will earn a 1% rate of return by the end of the year. But, if you use the money to pay down your car loan you will be saving yourself 5% this year. So, the opportunity cost of keeping the money in your savings account is roughly 5% - what you could have saved yourself by paying down your loan. If you use the money to pay down your loan, the opportunity cost is the 1% that you could have earned in your savings account. Which would you rather do, 1) earn 1% or 2) save 5%? Another way to ask this question is, "which option has the **lowest** opportunity cost?" You always want to use your money (or any asset) so that you have the lowest opportunity cost. That is, you want to use your money so that you give up the least amount possible. An easy way to remember this is that Opportunity Cost is a cost, and you always want to have the lowest costs possible.

Please note – it is hard to put a dollar value or percent return on many of the opportunities that you have for your money. For example, let's look at two possibilities. Assume that you received $1,000 from your grandparents. But in the first situation you have no money in your savings account. If you have a financial emergency (car breaks down, computer dies, etc.) you might have to get an emergency loan (possibly at a high interest rate) – and you don't like to owe money to others. In the second situation you already have $2,000 in your savings account, enough to cover most of your financial emergencies. This means that you can sleep much easier and you have fewer financial worries. It is hard to put a dollar value on what that "peace of mind" is worth from the first situation to the second situation.

So, RIO – Risk, Inflation, Opportunity Cost – are the main reasons that money is more valuable today than in the future. This is just another way of saying that you'd rather have a dollar in hand today rather than the promise of receiving a dollar in the future. That seems pretty simple. But why is this such a powerful force in the financial arena? The answer is "compound interest." Compounding is a term that indicates how often the interest that you earn on an investment is converted into principal. When the interest that you earn is converted into principal, your money begins to snowball. You are "earning interest on interest." Here's an example – assume that you invest that $1,000 you received from your grandparents into an account that will pay you 5% interest one time per year (we call this "annual compounding") at the end of each year. After 1 year your account will be worth $1,050:

$1,000 of principal invested into the account
5% Interest rate (rate of return)

Interest earned in the first year = $1,000 x 5% = $50

Account value = $1,000 of principal + $50 of interest = $1,050

Because you had $1,000 in your account and then you earned $50 of interest, your account is now worth $1,050. Here's where it gets exciting! From now on, the bank will treat that $50 of interest as principal. That means you will earn interest on the entire $1,050 this year, not just on the $1,000 you initially invested. So how much will your account be worth after the second year?

Interest earned in the second year = ($1,050) x 5% = $52.50

Account value = $1,050 +$52.50 = $1,102.50 after the second year

Notice that you earned more interest in the second year ($52.50 > $50) because of the compounding. I know, you're saying, "big whoop, it's only $2.50 more. Who cares?" Here's why you should care – let's leave that money in the account for the next 40 years. If it earns 5% each year your initial $1,000 will grow to over $7,000!! ($7,040 to be exact). Your money increased by 600% over that 40 years, and all you had to do was sit there and watch it grow. If you could have earned an annual return of 10%, your $1,000 would have grown to more than $45,250 over the 40-year period. If that's not worth a "big whoop", I don't know what is!

Here's another reason that time value of money is such a powerful force. Let's assume that you invest $500 each year into an account that will earn a return of 8%, compounded annually (one time per year). After 5 years your account will be worth $2,933 – you've invested $2,500 ($500/year x 5 years = $2,500), so you have earned $433 interest over that time. Now assume that you invest $500/year for the next 50 years. You will have invested $25,000 of principal over that time, but your account will have grown to over $286,000 due to compound interest!! Yes, you read that correctly; your $25,000 grew to over $286,000 over that 50-year period. And it's all due to compound interest.

If you are good at math, you will see that compound interest grows exponentially each year. If you aren't good at math, think of it this way – think of a jet airplane taking off from the runway. At first it climbs very slowly (like the $2.50 increase in interest from above). But as it gains some speed it starts to shoot up into the sky (like the $286,000!). That is how compound interest works – slow growth for the first 10-15 years, and then it really takes off!

There are a lot of different calculations that we can do with time value of money. We can look at what our money in hand today will grow to in the future. Or we can look at how much we would rather have in hand today instead of waiting to receive money in the future. And lots of other different variations. You already know how to calculate loan payments – that is also a time value of money calculation. But let's just focus on three main uses of time value of money:

Future Value (FV) of a Lump Sum
Present Value (PV) of a Lump Sum
Future Value of an Annuity (FVA)

Wait a minute, there are some terms here that we haven't defined yet. Don't worry, here are the definitions:

Future Value (FV) = what the money you have today will be worth in the future

Present Value (PV) = how much money you would rather have today (the present) instead of waiting to receive money sometime in the future

Lump Sum = a one-time investment; money only changes hands one time

Annuity = a stream of constant payments over a period of time (like loan payments)

There are several ways to calculate time value of money problems:

- Time Value of Money tables (just like the loan factor tables)
- Spreadsheets
- Financial calculators
- Time Value of Money formulas

We are going to focus on using the tables and a spreadsheet. For both methods it is helpful to develop a chart of information. This will help you identify the information that you know, and it will also help you determine what you are solving for.

Future Value of a Lump Sum

Like the name says, we use Future Value of a Lump Sum to determine how much the money you invest today (and today only) will be worth in the future. The example of calculating what your $1,000 investment today (and today only) would grow to after 40 years is a Future Value of a Lump Sum problem. We use Table 1 Future Value of a Lump Sum for these calculations. The tables are set up exactly the same as the annual loan payment table you used in the previous module – find the column with your interest rate (rate of return) and the row with the number of periods to determine the FV factor. The easiest way to learn is to work some problems.

Example 1. George wants to invest $5,000 in an account that will earn a 6% annual rate of return (interest rate). How much will he have in his account 10 years from now? First, let's build a chart of the information:

N = Number of periods
I = Interest rate
PV = Present Value, or what you are investing today (lump sum)
PMT = Payment, or how much you will be investing or receiving every period (annuity)
FV = Future Value, or what your account will be worth in the future (lump sum)

N = 10 years
I = 6% per year (this needs to be in the same units as N. In this case, both are in "years")

PV = $5,000 lump sum invested today
PMT = $0 (George is not making or receiving payments each year other than the interest)
FV = ?? This is what we are solving for

Using Table 1, the FV factor for 6% for 10 years is 1.7908. We calculate the FV of the lump sum by multiplying the PV by the factor:

FV of Lump Sum = $5,000 x 1.7908 = $8,954

George's $5,000 investment will grow to $8,954. He will earn interest worth $3,954 ($8,954 - $5,000) over this 10-year period.

To solve this problem using the spreadsheet, open the Lesson 8 TVM Calculator spreadsheet and click on the "FV Calculator" worksheet (at the bottom of the screen). Then, you simply enter the information from your chart into the spreadsheet.

Number of Years (N) = 10
Annual Rate (I) = 6%
Present Value (PV Lump Sum) = $5,000
Payment to be Made (PMT or Annuity) = $0

The END or BGN (0/1) is not needed for lump sum calculations; you only need to worry about this for PMT problems. END means that the payments are made at the end of the period. BGN means that the payments are made at the beginning of the period (starting today).

When you enter the information into the spreadsheet you will see the Future Value is automatically calculated. The result is $8,954.24. The spreadsheet will give you more accurate answers than the tables because the tables round off the factors to 4 decimal places; the spreadsheet does not round the factors at all.

The nice part of using the spreadsheet is that you can change one factor at a time to see how your answer will change. Change N to 40 years to see what George's account will be worth at the end of 40 years. Wow! It'll be worth $51,428.59

Example 2. Let's look at the impact of inflation. Current gasoline prices are around $2.00/gal. What will the price of gas be in 15 years if prices increase (inflate) at a rate of 4% each year?

N = 15 years
I = 4% per year
PV = $2.00 (current price of gas)
PMT = $0 (we aren't making any other payments)
FV = ??

Using Table 1, we can see the FV factor for 4% and 15 years is 1.8009. So we can estimate the future gas price to be:

FV of Gas Price = \$2.00/gal x 1.8009 = \$3.60/gal in 15 years

Present Value of a Lump Sum

This is the opposite of FV of a Lump Sum. The only difference is that we will use Table 2 or the "PV Calculator" of the spreadsheet. PV of a Lump Sum calculates how much money you would rather have in hand today instead of waiting to receive money in the future. Another way to use PV of a lump sum is to determine how much you need to invest today to reach a certain goal (for example, a down payment on a house). This is also an excellent way of determining the maximum you should be willing to pay for something today.

Example 3. You want to have \$20,000 in an account for a down payment on a house at the end of 10 years. Your account earns 5% annually. How much do you need to invest today to reach your goal of \$20,000 after 10 years?

N = 10 years
I = 5% per year (the annual inflation rate)
PV = ?? (this is what we are solving for)
PMT = \$0 (we aren't making any other payments)
FV = \$20,000 (what we want to have)

Using Table 2 we see that the PV factor for 5% for 10 years is 0.6139. To calculate how much we have to invest today to reach our goal, simply multiply the FV by the factor:

PV to Invest Today = \$20,000 x 0.6139 = \$12,278

If you were to invest \$12,278 today, it would grow to \$20,000 after 10 years.

Example 4. Mary thinks that a certain piece of land will be worth \$10,000/acre at the end of 5 years. She wants to purchase the land, but only if she can earn a profit (return) of 8% per year. What is the maximum that Mary can afford to pay for this land today?

N = 5 years
I = 8% per year (the annual rate of return she wants to earn)
PV = ??
PMT = \$0
FV = \$10,000 (what Mary expects the land to be worth)

Using Table 2, the factor for 8% for 5 years is 0.6806. This means that the maximum price that Mary can afford to pay for the land is:

Maximum Purchase Price = \$10,000 x 0.6806 = \$6,806/acre

Mary can earn an 8% rate of return on this land if she pays less than \$6,806/acre. Pretty neat, huh?! Now, use the spreadsheet to determine the maximum purchase price if Mary wants to earn a 5% rate of return.

You can always double-check your answers in time value of money problems by working backwards. For example, if you are working a Present Value question, you can use Future Value to check your answer (and the other way around!). Let's do this for Mary's example. We have calculated that the maximum purchase price (the PV) is \$6,806/acre if she wants to earn an 8% return. To double-check this answer, let's use the \$6,806 as the PV and solve for the FV.

N = 5
I = 8%
PV = \$6,806 (the PV you just calculated)
PMT = \$0
FV = ?? (your answer should be very close to \$10,000, the FV from the original situation)

The FV factor for 8% for 5 years is 1.4693. So the future value of the property should be:

FV = \$6,806 x 1.4693 = \$10,000 It works!!

Future Value of an Annuity
An annuity is a constant payment over a certain number of years. A great example would be investing money for a down payment on a house, or for your retirement. The difference between annuities (payments) and lump sums is that with annuities you are making (or receiving) a payment every year; with lump sums you are only investing money one time. The process of calculating the Future Value of an Annuity (FVA) is exactly the same as above. Simply multiply the annual payment you are making by the FVA factor. Here are some examples:

Example 5. Greta's business has been successful, so she has money that she wants to invest for her retirement. She thinks she can invest \$5,000/year for the next 30 years. Her investment should earn an annual return of 7%. How much money will Greta have in her retirement account at the end of 30 years?

N = 30 years
I = 7% per year
PV = \$0 (there is no lump sum, only payments)
PMT = \$5,000 per year
FV = ??

Use Table 6 Future Value of Annuity Factors to get the correct factor. For 7% over 30 years the factor is 101.0730. So, if Greta invests \$5,000/year for the next 30 years her retirement account will have:

FVA = $5,000 x 101.0730 = $505,365

This can't be correct, can it? She only invested $150,000 ($5,000/year x 30 years). How can it grow to over $500,000? The answer is "compound interest". And an important thing for you to remember is that "time is on your side." The earlier you can start investing for a goal, the less money you actually need to invest!

Here's a good place to talk about the END and BGN cells in the TVM Calculator spreadsheet. END indicates that you wait until the end of the period to make your payments. Rather than making the first payment today you will wait until the end of the year to make that first payment. BGN (Beginning) indicates that you will start making payments today, and at the beginning of every period. You will end up with more money over time if you make your payments at the beginning of each period because you will have one more period of compound interest on your side. Just so you know, the Future Value of Annuity Factor table (Table 6) assumes that all payments are made at the beginning of each period.

Use the spreadsheet to see how big of a difference it will make if Greta makes either END or BGN payments. Here's how you do it. Let's start with BGN payments:

N = 30 years
I = 7% per year
PV = $0
PMT = $5,000 per year
END or BGN = 1 (use 1 for BGN payments; use 0 for END payments)
FV = $505,365.21 (the same answer we got using factor)

To look at END payments, simply change the END or BGN cell to 0 (zero).

N = 30 years
I = 7% per year
PV = $0
PMT = $5,000 per year
END or BGN = 0 (use 1 for BGN payments; use 0 for END payments)
FV = $472,303.93 (the same answer we got using factor)

Now you can see that Greta will have $472,303.93 in her retirement account at the end of 30 years – a difference of more than $33,000 just because Greta waited one year to start investing! Do you see why it is important to start investing today rather than waiting?!

There are lots of other calculations we can do with Time Value of Money, but these examples (along with calculating loan payments) are the most common ones. TVM is a powerful concept that will come in useful to a business manager in many ways!